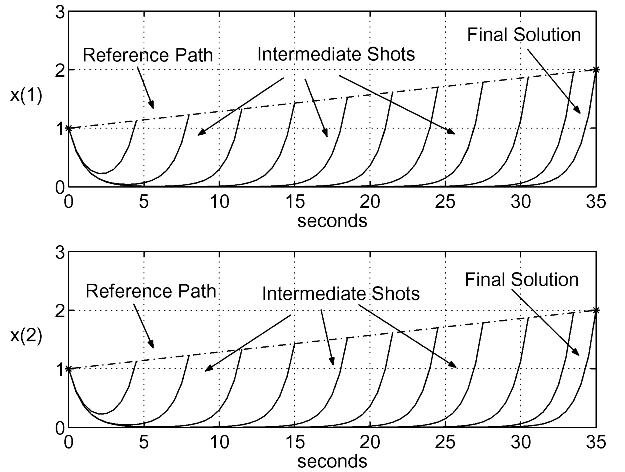

Fig. 2 MSSM applied to the linear system example.

for the MSSM, MSM, and CM was the classical fourth-order Runge–Kutta method. Figure 2 shows the result of the MSSM including the reference path and the intermediate trajectories. The SSM failed for this problem (with $t_f = 35$), while both the MSM and MSSM were successful.

The collocation method tested was the bvp4c routine in MATLAB®. For this method the mesh selection is based on the residual of the C^1 continuous solution that is fourth-order accurate uniformly in the time interval.[10] For $t_f = 1$ we chose 101 uniformly spaced nodes corresponding to a time step of 0.01 s as the initial mesh. For $t_f = 35$ a uniform mesh corresponding to a time step of 0.004 s was chosen at the initial step.

Conclusions

A new method for solving two-point boundary-value problems has been presented. An example was provided that clearly illustrates that the MSSM results in an accurate solution that takes significantly less computation time than the MSM, finite difference, and collocation methods.

Among its desirable features are that it requires the inversion of much smaller matrices than those required to be inverted in the MSM, finite difference, and collocation methods. Another fact that makes the MSSM more appealing is that the solution results in a trajectory that satisfies the system differential equations. This property is very important in optimal control problems where the systems are unstable in forward time. The MSM, finite difference, and collocation methods do not share this property with the MSSM. Because of the instability of many systems in the forward direction, these other methods can lead to erroneous solutions, in the sense that the solution trajectory on reintegration does not satisfy the boundary conditions at the final time.

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Approximate Analytical Criterion for Aircraft Wing Rock Onset

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I. Introduction

THERE is continuing interest in analytical criteria for aircraft departure because of the physical insight they provide into the associated instability phenomena and their use in identifying critical aerodynamic parameters related to departure susceptibility. However, exact analytical criteria are usually too complicated to be of any use; hence, considerable effort has been made over the years to arrive at approximate analytical criteria for departure phenomena that may be usefully employed early in the aircraft design process. Initial work by Bryant and Zimmerman (see Ref. 1) established approximate criteria for static directional instability, also called directional divergence or yaw departure. Dynamic directional instability, on the other hand, is associated with unstable Dutch roll oscillations linked to onset of a phenomenon called wing rock.[2] An exact analytical criterion for wing rock onset, based on the vanishing of the Routh discriminant, is attributed to Duncan[3]; however, the resulting expression is too unwieldy to provide any physical insight and, not surprisingly, has found little use. [The Routh discriminant is given by $R = D(BC - D) - EB^2 = 0$, where B, C, D, and E are the coefficients of the lateral–directional characteristic polynomial, $P_{\text{lat}}(\lambda) = \lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E$.] Instead, Moul and Paulson[4] came up with an approximate criterion called "dynamic directional instability parameter" or $C_{n_{\beta\text{dyn}}}$, but attempts[5,6] to relate rigorously the $C_{n_{\beta\text{dyn}}}$ parameter to an instability phenomenon led to the conclusion that $C_{n_{\beta\text{dyn}}}$ was, in fact, only an approximate static directional instability criterion. Efforts to derive a useful, approximate version of the Routh discriminant criterion for wing rock onset have been ongoing (e.g., as in Ref. 7), whereas, $C_{n_{\beta\text{dyn}}}$, by virtue of being an important component of the Routh discriminant, has de facto found

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wide use as an approximate criterion for dynamic directional departure as well. On a different track, researchers[8–10] have attempted to derive improved literal approximations to the Dutch roll damping, with the condition of zero Dutch roll damping providing an approximate criterion for wing rock onset. These literal approximations are generally based on the premise that there exist distinct well-damped roll and lightly damped spiral modes in the lateral dynamics, but at the moderately high values of angle of attack typical of wing rock onset, the effect of decreased roll damping and increased dihedral stability commonly gives rise to an oscillatory, coupled roll-spiral mode with light damping, also called the lateral phugoid.[11] Then, at the point of onset of wing rock, the lateral–directional dynamics typically consists of a lightly damped lateral phugoid mode and a Dutch roll mode with zero damping; consequently, instability criteria based on the literal approximations are rendered invalid.

The aim of the present Note is to derive rigorously an approximate analytical criterion for wing rock onset, allowing for the presence of a lightly damped lateral phugoid mode. This derivation follows a novel approach that is not based on either approximations of the coefficients of the characteristic polynomial or on the literal approximations. The criterion derived here shows, for the first time, an explicit link between the $C_{n_{\beta\mathrm{dyn}}}$ parameter and wing rock onset that has been elusive for nearly half a century.

II. Approximation by Hamiltonian System

Consider the linearized lateral–directional dynamics equations, for small perturbations about a straight and level trim flight, in the second-order form introduced in Ref. 10. These equations, in the perturbation variables, $\Delta y_{\mathrm{lat}} = [\Delta\beta, \Delta\mu]$, appear as follows:

$$\Delta\ddot{y}_{\mathrm{lat}} + [C_{\mathrm{lat}}]\Delta\dot{y}_{\mathrm{lat}} + [K_{\mathrm{lat}}]\Delta y_{\mathrm{lat}} = 0 \tag{1}$$

where C_{lat} and K_{lat} are the 2×2 damping and stiffness matrices, respectively, for the lateral–directional dynamics. The complete equations in $\Delta\beta$ and $\Delta\mu$ are as follows[10]:

$$\begin{aligned}\Delta\ddot{\beta} &+ [-(N_r'\cos\alpha_0 - N_p'\sin\alpha_0) - (Y_\beta/V_0)]\Delta\dot{\beta}\\ &+ [(N_r'\sin\alpha_0 + N_p'\cos\alpha_0) - (g/V_0)]\Delta\dot{\mu}\\ &+ [N_\beta' + (Y_\beta/V_0)(N_r'\cos\alpha_0 - N_p'\sin\alpha_0)]\Delta\beta\\ &+ [(g/V_0)(N_r'\cos\alpha_0 - N_p'\sin\alpha_0)]\Delta\mu = 0\end{aligned} \tag{2}$$

$$\begin{aligned}\Delta\ddot{\mu} &+ [(L_r'\cos\alpha_0 - L_p'\sin\alpha_0)]\Delta\dot{\beta} - [(L_r'\sin\alpha_0 + L_p'\cos\alpha_0)]\Delta\dot{\mu}\\ &- [L_\beta' + (Y_\beta/V_0)(L_r'\cos\alpha_0 - L_p'\sin\alpha_0)]\Delta\beta\\ &- [(g/V_0)(L_r'\cos\alpha_0 - L_p'\sin\alpha_0)]\Delta\mu = 0\end{aligned} \tag{3}$$

where the primed derivatives are defined as $N_\beta' = N_\beta\cos\alpha_0 - L_\beta\sin\alpha_0$, and so on, the dimensional stability derivatives have their usual meanings (e.g., as in Ref. 12), and the subscript 0 refers to the trim state. For the case of lateral–directional dynamics with only light damping, it is reasonable to approximate the original system, Eq. (1), by the following undamped approximation, obtained by ignoring the terms in the damping matrix C_{lat}:

$$\Delta\ddot{y}_{\mathrm{lat}} + [K_{\mathrm{lat}}]\,\Delta y_{\mathrm{lat}} = 0 \tag{4}$$

[In the present context, it is adequate to consider the lateral–directional dynamics, Eq. (1), to be lightly damped when the roll and spiral modes have been replaced by the lateral phugoid; the error incurred due to this approximation at the point of wing rock onset is directly related to the damping of the lateral phugoid mode, which is usually quite small.] However, all of the terms in the $2\times 2 K_{\mathrm{lat}}$ matrix are retained, and they appear as follows:

$$K_{\mathrm{lat}} = \begin{bmatrix} N_\beta' + (Y_\beta/V_0)(N_r'\cos\alpha_0 - N_p'\sin\alpha_0) & (g/V_0)(N_r'\cos\alpha_0 - N_p'\sin\alpha_0)\\ -L_\beta' - (Y_\beta/V_0)(L_r'\cos\alpha_0 - L_p'\sin\alpha_0) & -(g/V_0)(L_r'\cos\alpha_0 - L_p'\sin\alpha_0)\end{bmatrix} \tag{5}$$

Table 1 Eigenvalues of Hamiltonian system

Case	τ	$\lambda = \pm\sqrt{\tau}$	Stability
1	Both real negative	Two pairs of complex conjugate eigenvalues on the imaginary axis; $\pm i\omega_1$, $\pm i\omega_2$	Stable
2	Both real positive	Two pairs of real eigenvalues; $\pm\sigma_1$, $\pm\sigma_2$	Unstable
3	Both real, one positive, one negative	One pair of complex conjugate eigenvalues on the imaginary axis, another pair of real eigenvalues; $\pm i\omega$, $\pm\sigma$	Unstable
4	Complex conjugate pair	Two pairs of complex conjugate eigenvalues; $\pm(\sigma \pm i\omega)$, $\sigma \neq 0$	Unstable

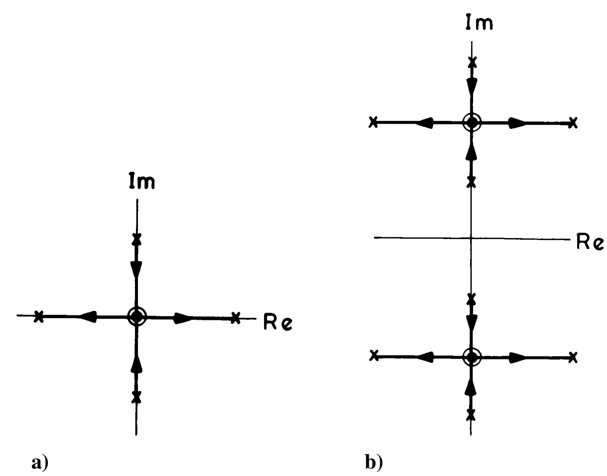

Fig. 1 Instability mechanisms for Hamiltonian systems: a) static and b) dynamic.

Note that the traditional practice of considering an undamped approximation to the aircraft dynamics equations by setting the aerodynamic damping terms to zero (as in Ref. 6 and references therein) is flawed because it does not render the resulting system conservative; worse, several legitimate terms in the stiffness matrix K_{lat} also get dropped in the process. On the other hand, the undamped approximation [Eq. (4)], obtained by ignoring the C_{lat} matrix, can be shown[13] to have a Hamiltonian structure and, hence, is naturally conservative, but, at the same time, features all of the aerodynamic damping derivatives, N_r, L_p, N_p, and L_r, by virtue of their appearance in the K_{lat} matrix [Eq. (5)].

The undamped dynamics [Eq. (4)], being a Hamiltonian system, is known to have eigenvalues λ that are constrained to be symmetric about the origin in the complex plane.[14] The eigenvalues are given by solutions of a fourth-order characteristic equation in λ, or an equivalent second-order equation in $\tau = \lambda^2$, as follows:

$$\left|\lambda^2 I + K_{\mathrm{lat}}\right| = 0 \qquad \text{or} \qquad |\tau I - (-K_{\mathrm{lat}})| = 0 \tag{6}$$

where I is the 2×2 identity matrix. For the 2×2 matrix $-K_{\mathrm{lat}}$, there are only two possibilities for the roots τ of its characteristic equation, namely, a complex conjugate pair or two real roots, thus giving rise to the four possible arrangements listed in Table 1 for the eigenvalues λ of the Hamiltonian system [Eq. (4)]. Note from Table 1 that there is only one arrangement of eigenvalues for which the Hamiltonian system is stable, that is, when all of the eigenvalues lie on the imaginary axis.

III. Instability Criteria

There are two standard mechanisms (Fig. 1) by which the stable Hamiltonian system in Eq. (4) with two pairs of complex conjugate eigenvalues on the imaginary axis (case 1 of Table 1) can become unstable: 1) static and 2) dynamic.

A. Static Instability

In this mechanism, one pair of eigenvalues on the imaginary axis moves toward the origin on the complex plane, as shown in Fig. 1a. The eigenvalues collide at the origin and separate along the real axis as a symmetric pair, which is the unstable case 3 of Table 1. Thus, the point of onset of static instability corresponds to the presence of multiple (two) eigenvalues λ at the origin, which is the same as one zero root τ of the characteristic equation (6) for the K_{lat} matrix. This condition is given by the determinant of the K_{lat} matrix being zero, that is, $\det(K_{\text{lat}}) = 0$, and the resulting static instability criterion, called S_{lat}, appears as follows:

$$S_{\text{lat}} = (g/V_0)[(L'_\beta N'_r - N'_\beta L'_r)\cos\alpha_0 - (L'_\beta N'_p - N'_\beta L'_p)\sin\alpha_0] = 0 \tag{7}$$

The condition $S_{\text{lat}} = 0$ is an exact criterion for static instability in the spiral mode: exact because the static instability mechanism is independent of the presence or absence of damping effects.

B. Dynamic Instability

In this case (Fig. 1b), two pairs of complex conjugate eigenvalues $(\pm i\omega_1, \pm i\omega_2)$ approach each other and collide on the imaginary axis. They then separate out as two complex conjugate pairs, one each in the right and left half-planes, symmetric about the origin, which is case 4 in Table 1. Thus, onset of dynamic instability corresponds to the condition of repeated eigenvalues on the imaginary axis, that is, $\omega_1 = \omega_2$, which happens to be a resonance condition. [The dynamic instability mechanism has been variedly referred to in the literature as a $1 : -1$ nonsemisimple resonance mechanism or a Hamiltonian Hopf bifurcation (see Ref. 14)]. The requirement of resonance can be seen from case 1 of Table 1 to be the same as the condition of repeated roots τ of the characteristic equation (6), that is, $\tau_1 = \tau_2 < 0$, which is given by the discriminant of the quadratic equation (6) in τ being zero. After some algebraic manipulation, this gives the following criterion, called D_{lat}, for the onset of dynamic instability in the Hamiltonian system [Eq. (4)]:

$$\begin{aligned} D_{\text{lat}} = {} & [N'_\beta + (Y_\beta/V_0)(N'_r\cos\alpha_0 - N'_p\sin\alpha_0) \\ & - (g/V_0)(L'_r\cos\alpha_0 - L'_p\sin\alpha_0)]^2 - 4S_{\text{lat}} = 0 \end{aligned} \tag{8}$$

where S_{lat} is the expression for the static instability criterion in Eq. (7). The condition $D_{\text{lat}} = 0$ is an approximate criterion for dynamic directional instability, that is, wing rock onset. (The Hamiltonian Hopf bifurcation of the undamped system [Eq. (4)] thus, serves as an approximation to the Hopf bifurcation (see Ref. 15) for onset of wing rock in the original damped system [Eq. (1)].)

C. Numerical Illustration

The effectiveness of the D_{lat} condition as an approximate analytical criterion for wing rock onset is illustrated with data for an example aircraft taken from Ref. 16. For this data set, the exact condition for onset of dynamic directional instability in level flight has previously been numerically computed using a bifurcation procedure.[17] Both the bifurcation analysis in Ref. 17 and independently obtained root locus plots for the complete lateral–directional dynamics in Ref. 10 have indicated that the Dutch roll mode undergoes a Hopf bifurcation at a trim angle of attack of 22.5 deg, which marks the onset of wing rock. The variation of the quantity D_{lat} for the same aircraft data is plotted in Fig. 2 as a function of trim angle of attack. The approximate criterion in Eq. (8), marked by the zero crossing

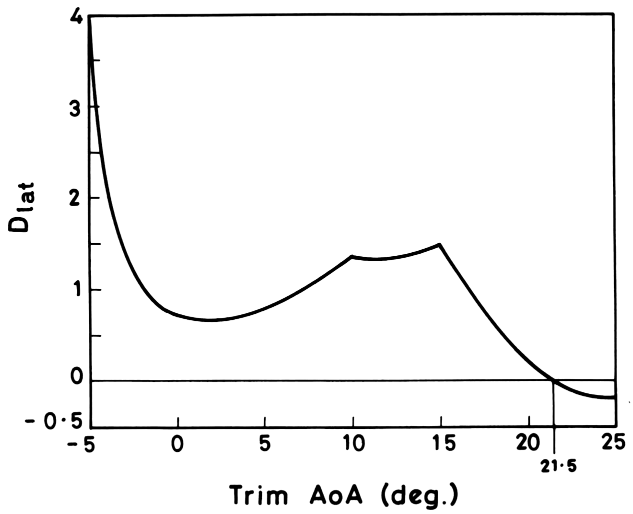

Fig. 2 Quantity D_{lat} for trim AOA varying between −5 and 25 deg predicting onset of wing rock at 21.5-deg AOA.

of the quantity D_{lat} in Fig. 2, gives a trim angle of attack of 21.5 deg for onset of wing rock, which is within 1 deg of the actual value. The instability mechanism corresponding to the zero crossing of the D_{lat} criterion in Fig. 2 can be confirmed to be a Hamiltonian Hopf bifurcation by plotting the locus of eigenvalues for the undamped lateral–directional dynamics [Eq. (4)], with varying angle of attack (AOA), as shown in Ref. 13.

IV. Relation to $C_{n_{\beta\text{dyn}}}$

A nondimensional version of the D_{lat} criterion can be obtained by factoring out $(\bar{q}Sb/I_z)^2$ from Eq. (8), as follows:

$$\begin{aligned} & \left\{ C'_{n_\beta} + \left(\frac{g}{V_0}\right)\left(\frac{b}{2V_0}\right)\left[\frac{C_{y_\beta}}{C_{L_0}}\left(C'_{n_r}\cos\alpha_0 - C'_{n_p}\sin\alpha_0\right)\right.\right. \\ & \left.\left. + \frac{I_z}{I_x}\left(C'_{l_r}\cos\alpha_0 - C'_{l_p}\sin\alpha_0\right)\right]\right\}^2 \\ & - 4\left(\frac{g}{V_0}\right)\left(\frac{b}{2V_0}\right)\left(\frac{I_z}{I_x}\right)\left[\left(C'_{l_\beta}C'_{n_r} - C'_{n_\beta}C'_{l_r}\right)\cos\alpha_0\right. \\ & \left. - \left(C'_{l_\beta}C'_{n_p} - C'_{n_\beta}C'_{l_p}\right)\sin\alpha_0\right] = 0 \end{aligned} \tag{9}$$

Notice that the leading term in Eq. (9) is C'_{n_β}, which, when the primed derivative is replaced with unprimed ones, can be written as $C_{n_\beta}\cos\alpha_0 - (I_z/I_x)C_{l_\beta}\sin\alpha_0$, which is precisely the $C_{n_{\beta\text{dyn}}}$ parameter introduced by Moul and Paulson.[4] Remarkably, this is the first instance where the $C_{n_{\beta\text{dyn}}}$ parameter has been explicitly linked with a dynamic instability phenomenon, namely, wing rock onset.

Also note that all of the aerodynamic damping derivatives, which have been retained in this derivation, do appear as part of the D_{lat} criterion in Eq. (8) or its nondimensional version in Eq. (9). If the aerodynamic damping terms were to be ignored, as has been the practice in the past, the criterion in Eq. (9) would reduce to $C_{n_{\beta\text{dyn}}} = 0$, which has been recognized to be only an approximate static instability criterion.

Another point worthy of note is that the D_{lat} criterion has been derived without neglecting the effects of gravity. Under the assumption that gravity influences only the spiral mode dynamics, and the belief that the lateral–directional departure phenomena are entirely aerodynamic in nature, it has been a common mistake in the past to ignore the gravity terms a priori while attempting to derive analytical criteria for directional instability phenomena.[6] The derivation of the D_{lat} criterion in the present Note, however, clearly shows that terms involving gravity do have an important role to play in the dynamic directional instability phenomenon of wing rock onset. In fact, when the effect of gravity is neglected by allowing the factor (g/V_0) to go to zero in Eq. (9), the approximate criterion for wing rock onset reduces to $C_{n_{\beta\mathrm{dyn}}} = 0$, which, as seen before, merely signifies static instability.

V. Conclusions

An approximate analytical criterion for dynamic directional instability, called D_{lat}, has been derived that provides a useful and simpler alternative to the exact Routh discriminant condition for wing rock onset. The D_{lat} criterion reveals, for the first time, an explicit link between wing rock onset, a dynamic instability phenomenon, and the $C_{n_{\beta\mathrm{dyn}}}$ parameter.

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Learning-Based Sensor Validation Scheme Within Flight-Control Laws

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I. Introduction

RESEARCH on analytical redundancy (AR) and model-based sensor fault detection identification and accomodation (SFDIA) has shown that AR[1] can provide fault-tolerance capabilities for aircraft where physical redundancy in the onboard sensors is not a cost-effective solution. Although effective schemes for linear-time-invariant systems[2] are already established, the extension of these schemes to nonlinear systems still presents substantial research challenges.[1] In Ref. 3 three of the authors proposed a sensor fault detection isolation accomodation (SFDIA) scheme based on neural-network (NN) approximators. In this effort this scheme has been improved through a new fault-identification logic. Failures were simulated in closed-loop conditions, and a detailed analysis was performed to determine the smallest detectable and identifiable faults along with the relative detection and isolation delay. A new hybrid configuration for the neural approximators was also introduced.

II. Analytical Redundancy-Based SFDIA Scheme for the Longitudinal Aircraft Dynamics

The main variables involved in the aircraft longitudinal dynamics are angle of attack α (rad), pitch rate q (rad/s), normal acceleration $a_z(g)$, elevator deflection δ_e (rad), airspeed V (m/s), and altitude $H(m)$. The AR between these signals is embedded in the equations of motion of the aircraft through the normal force equation:

$$ma_z = \rho(H)V^2 SC_z(\alpha, \delta_e, V, q/V)\big/2 \tag{1}$$

The AR between the variables in Eq. (1) was used to implement the nonlinear input/output models required in the SFDIA scheme. In particular, the following estimation models were developed:

$$\hat{\alpha}(k) = f_\alpha\big(V(k), H(k), a_z(k)\big/V^2(k), q(k)/V(k), \delta_e(k)\big) \tag{2}$$

$$\hat{q}(k) = f_q\big(V(k), H(k), V(k), a_z(k)\big/V^2(k), \alpha(k), \delta_e(k)\big) \tag{3}$$

$$\hat{a}_z(k) = f_{a_z}\big(V(k), H(k), V(k), q(k)/V(k), \alpha(k), \delta_e(k)\big) \tag{4}$$

$$\hat{\delta}_e(k) = f_{\delta_e}\big(V(k), H(k), a_z(k)\big/V^2(k), \alpha(k), q(k)/V(k)\big) \tag{5}$$

In the preceding equations, k indicates the current time instant. The use of q/V and a_z/V^2 in lieu of a_z and q as inputs allows the keeping

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